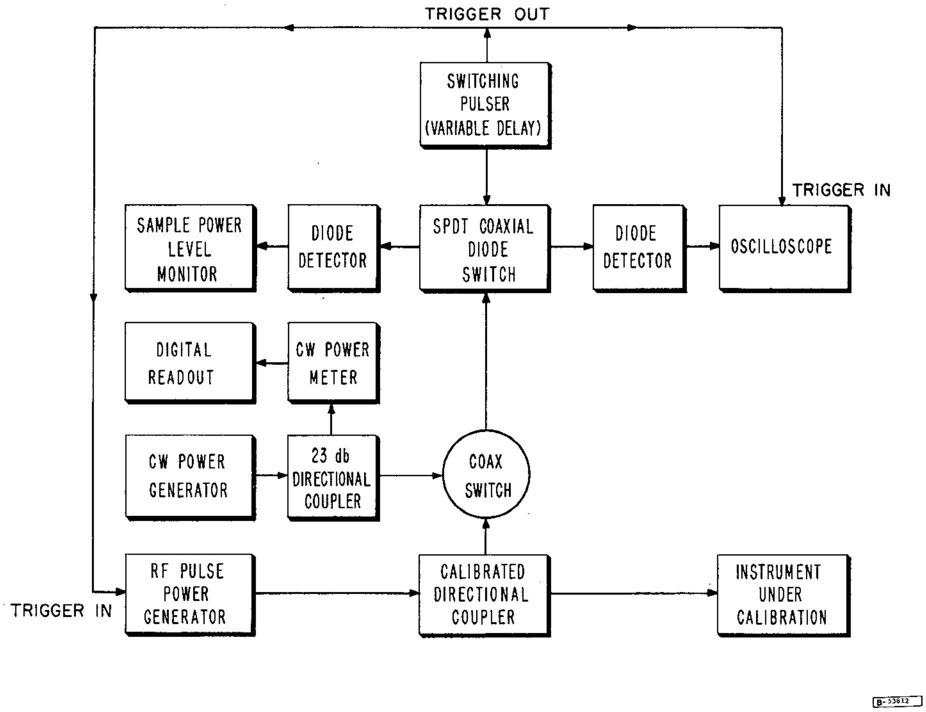

Fig. 1. Block diagram of sampling comparison method used in making RF peak-pulse power calibrations. Equal samples of pulsed RF power and CW power are intercompared and the peak pulse power computed by an accurate measurement of the CW signal.

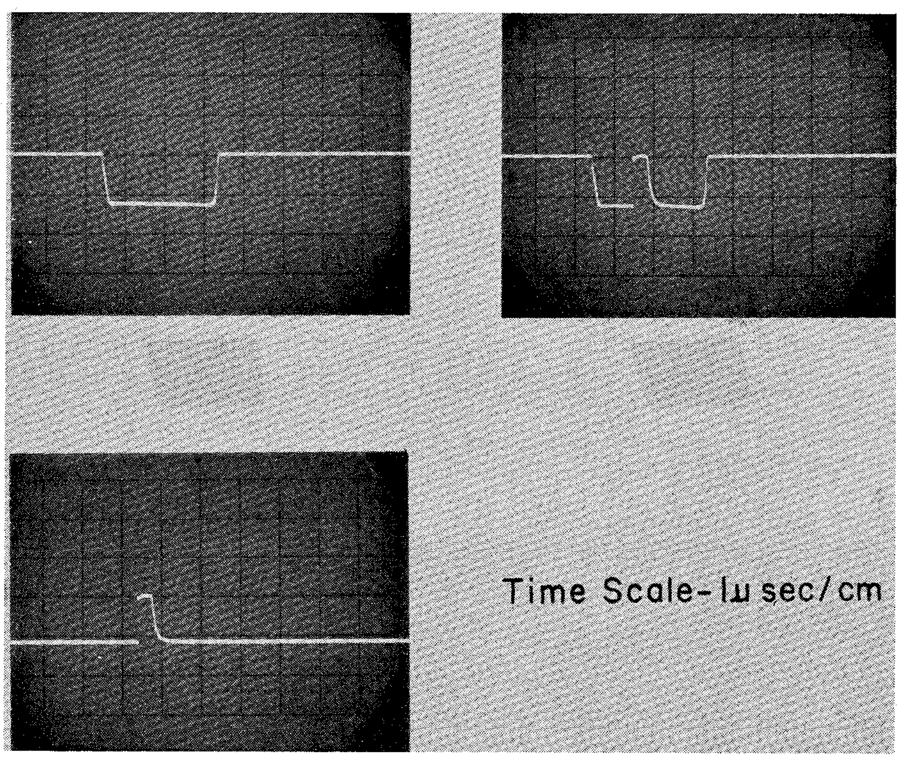

Fig. 2. Typical waveforms observed on oscilloscope when connected to normally closed arm of the coaxial diode switch. (a) Envelope of unsampled RF pulse. (b) Envelope of sampled RF pulse. (c) Envelope of sampled CW signal.

power from an RF pulse for measurement. The switch is also used to obtain a similar sample from a CW signal whose power can be accurately measured. By adjusting the CW level to obtain similar readings (as shown in Fig. 2) for the pulse and CW samples on a level monitor, the peak-pulse power can be determined.

While the range of the basic system is limited to 0.1 to 1.2 watts by the power handling capability of the diode switch, higher or lower powers can be measured by means of calibrated directional couplers.

The performance of the calibration console was checked by comparing it against the peak-pulse power standard maintained by the High Frequency Electrical Standards Section. Since the calibration of the directional couplers is a possible additional source of error, intercomparisons of the two systems were first made over the power range of 0.1 to 1.2 watts without these couplers, then at higher power levels with the couplers. The results show no significant difference between the two standards and both, therefore, have the same degree of uncertainty, ±3 percent. This is based on an estimate of 2 percent for the basic system which includes the measurement of the CW power, and an additional 1 percent for the calibration of the directional couplers.

It is planned to expand this calibration service to include additional frequency bands and higher power levels. Also, new type diode switches, which promise to give improved accuracy to the basic system, are under development at the Radio Standards Laboratory. It is hoped the uncertainty of measurement can be reduced to 1 percent or better.

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On the Impedance of a Finite Slot

It is well known that in boundary value problems which involve slots or apertures the analysis becomes easier when the assumption of infinitesmally narrow slots can be made. Doing so however, will usually result in divergent expressions for slot impedance and current across the slot. One ordinarily gets around this by stating that, since in a real or physical problem the slot width must be finite, the impedance of such a slot can be obtained by summing the first few terms of the infinitesimal slot series and throwing away the divergent part. It is the purpose of this correspondence to show that the impedance of a finite, but narrow slot can be reasonably approximated by the first N terms of the infinitesimal slot series

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where N is given by $N \lesssim 1/\delta$ and δ is the angular width of the slot in radians.

Consider the slot admittant $Y(0)$ for an infinitesimally thin slot which is placed along the equator of a perfectly conducting sphere of radius a [1]

$$Y(0) = \sum_{n}^{\infty} Y_n(0) = \frac{j\pi}{\eta} \sum_{n}^{\infty} \frac{2n+1}{n(n+1)} \cdot \frac{[P_n{}^1(0)]^2}{\dfrac{n}{ka} - \dfrac{H_{n-\frac{1}{2}}{}^{(2)}(ka)}{H_{n+\frac{1}{2}}{}^{(2)}(ka)}} \cdot \qquad (1)$$

The series does not converge since finite contributions to the imaginary part of Y are added forever. The admittance for a slot of finite gap width $a\delta$, where $\delta \ll 1$ can be written as [2]

$$Y(\delta) = \sum_{n}^{\infty} Y_n(\delta) = \frac{j\pi}{\delta^2\eta} \sum \frac{2n+1}{n(n+1)} \frac{\left[P_n\left(\cos\left[\dfrac{\pi}{2} + \dfrac{\delta}{2}\right]\right) - P_n\left(\cos\dfrac{\pi - \delta}{2}\right)\right]^2}{\dfrac{n}{ka} - \dfrac{H_{n-\frac{1}{2}}{}^{(2)}(ka)}{H_{n+\frac{1}{2}}{}^{(2)}(ka)}} \cdot \qquad (2)$$

It is seen that this expression is similar to $Y(0)$, except that the Legendre functions are those appropriate for a finite slot. We will now show that the first few terms of both series are approximately equal. This comes about since the P_n's show little variation for small δ in the range of $\pi/2 - \delta/2$ to $\pi/2 + \delta/2$ when n is not too large (recalling that n gives the number of zeros that the Legendre function goes through in the range of θ from 0 to π). Here we can expand

$$P_n\left\{\cos\frac{\pi \pm \delta}{2}\right\}$$

about $P_n(0)$ giving for

$$[P_n(+) - P_n(-)]^2 = [\delta P_n{}^1(0)]^2 \left(1 - \frac{n^2 + n - 1}{12}\delta^2 + \cdots\right). \qquad (3)$$

Ignoring the δ^2 terms in the preceding Taylor expansion and then substituting it in (2) we obtain an approximate series for $Y(\delta)$ whose first few terms are those of the infinitesimal slot expression $Y(0)$. This approximation is valid for the first terms but becomes unusable for the higher terms. Let us denote by N the upper bound for the summation index n for which this approximation is valid, i.e.,

$$Y(\delta) = \sum_{n=1}^{N} Y_n(0) + \cdots. \qquad (4)$$

The validity of the preceding Taylor expansion for P_n determines N. The zeros of the Legendre function are almost uniformly distributed in the range of θ from 0 to π. Thus the number of zeros within the range of δ should be approximately equal to $\delta n/\pi$. Now for the two-term Taylor expansion to be valid, the Legendre function must not vary much over the gap width. This can be stated in terms of the number of zeros which are included in the gap region as $\delta n/\pi < 1$. This condition ensures that the slot does not extend beyond the first maximum of $P_n(\cos\theta)$ on either side of $\theta = \pi/2$. Hence it follows that an upper bound for N can be taken as $N \approx 1/\delta$. This condition follows also immediately if in (3) the terms in the round bracket are not to differ from unity by more than ϵ, when ϵ is small, then we must have $n \lesssim \sqrt{12\epsilon}/\delta$. Hence for $n \le N$, $Y_n(\delta) \simeq Y_n(0)$.

To evaluate the rest of the series (2) whose terms will show more and more variation over the gap width as n gets larger, we can make use of the asymptotic expression for P_n as n becomes large. Again retaining first-order terms the remainder of the series for the admittance of a finite slot can then be written as

$$\text{Remainder } Y(\delta) = \frac{j4ka}{\delta^2\eta} \sum_{n=N+2}^{\infty} \frac{2n+1}{n^3(n+1)}(1 - \cos n\delta) = \sum Y_\infty(\delta) \qquad (5)$$

where the Hankel function terms in the denominator of Y are approximated for large n as n/ka. The terms in the remainder series converge as $1/n^3$, whereas the remaining terms in the series for the infinitesimal slot diverge. An estimate of the remainder (5) can be obtained by noting that

$$\sum_{n=N+2}^{\infty} Y_n(\delta) < M \sum_{n=N+2}^{\infty} 1/n^3 \qquad (6)$$

where M is a constant. The series of $1/n^3$ terms can be summed by converting to a contour integral with poles along the real axis, which can then be deformed to a path parallel to the imaginary axis. By changing variables the resulting integral can be estimated yielding

$$\sum_{n=N+2}^{\infty} 1/n^3 < C\frac{1}{(N+2)^2} \qquad (7)$$

where C is another constant. Thus the slot admittance can be written as

$$Y(\delta) = \sum_{n=1}^{N} Y_n(0) + O(1/N^2). \qquad 8)$$

In case the exact value of the remainder series is desired,

$$\sum_{n=1}^{\infty} Y_\infty(\delta)$$

can be evaluated in closed form [3]. The remainder (5) can then be obtained by subtracting the sum of the first N terms, and then can be used to approximate the exact remainder to (2).

Thus it can be concluded that the admittance of a finite slot can be reasonably approximated by the generally divergent expression for the admittance of an infinitesimally narrow slot by summing the series to the first N terms, where N is given by $N \approx 1/\delta$, and δ is the angular width of the slot in radians. The error created in leaving off the remaining terms is then of the order $O(1/N^2)$.

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References

[1] S. Ramo and J. R. Whinnery, *Fields and Waves in Modern Radio*. New York: Wiley, 1953, sec. 12.23.
[2] S. A. Schelkunoff, *Electromagnetic Waves*. New York: Van Nostrand, 1943, pp. 472–474.
[3] M. A. Plonus, "On the impedance of a finite slot," Radiation Lab., University of Michigan, Ann Arbor, Memorandum 5548-505-M, July 1963.

Propagation of the Quasi-TEM Mode in Ferrite-Filled Coaxial Line

Brodwin and Miller,[1] in discussing the propagation of the quasi-TEM mode in ferrite-filled coaxial line, were apparently unaware of an earlier comprehensive treatment of the subject.[2,3] For purpose of comparison, the notations in this correspondence correspond to those used by Brodwin and Miller.

In the earlier treatment, the Suhl and Walker approximation for parallel plane geometry was extended to coaxial geometry by requiring that the conditions

$$|S_1| R_2 \ll 1, \quad |S_2| R_2 \ll 1, \quad |S_1| R_1 \ll 1, \quad |S_2| R_1 \ll 1 \qquad (1)$$

be satisfied where R_2 and R_1 are the outer and inner radii, respectively, of the coaxial line.[4,5] If conditions (1) are substituted directly into the exact determinantal equation for the quasi-TEM mode in coaxial geometry,[4] the determinantal equation reduces to[5]

$$\gamma_0{}^2 = -\omega^2\epsilon\left[\frac{\mu^2 - k^2}{\mu}\right] \qquad (2)$$

for nontrivial values of S_1 and S_2.

For parallel plane geometry, (2) is known as the Suhl and Walker approximation to the propagation constant of the quasi-TEM mode. It can be shown by direct substitution into the exact determintal equation that (2) is valid in parallel plane

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[1] M. E. Brodwin and D. A. Miller, "Propagation of the quasi-TEM mode in ferrite-filled coaxial line," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 496–503, September 1964.

[2] T. Teragawa, M. M. Weiner, and W. D. Fitzgerald, "Ferrite feasibility study, 200–400 Mc/s," Chu Associates Interim Development Repts 1–8, Contract Nobsr-72586, U. S. Navy Department Index. NE-120-704-20, June 1956–January 1959.

[3] M. M. Weiner, R. Teragawa, and W. D. Fitzgerald, "UHF ferrite phase shift properties," *1959 Proc. Electronic Components Conference*, p. 26.

[4] R. Teragawa et al., *op. cit.*,[2] Interim Development Rept. 1, June–August 1956, pp. 6–12.

[5] ——, *op. cit.*,[2] Interim Development Rept. 5, June–August 1957, pp. 23–25.